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M. Conte and E. Ganssauge: SOME CALCULATIONS CONCERNING THE MEASUREMENT OF THE POLARIZATION OF μ-MESONS, USING MØLLER-SCATTERING IN THE MAGNETIZED IRON PLATES OF A SPARK CHAMBER.

Nota interna: n° 151
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CONTENTS

I. - INTRODUCTION

For measuring the polarization of μ-mesons an experiment was made by Alikhanov et al.[1], where the authors registered Moeller-scattering processes with great energy transfers from the incident muons to the knock-on electrons of magnetized iron. In that case electron showers were created. The measurement was done with cosmic rays using energies from 3 to 6.5 BeV.

This method does not work in the region of low μ-energies, for in that case the scattered electrons cannot escape a thick scatterer and thus cannot be registered. On the other hand the counting rate increases with the thickness of the scatterer.

Therefore, following the idea of Stoppini, another method should be discussed, where the scattering takes place in the plates of a spark chamber. There the whole thickness of scattering material can be made rather thick for getting a big counting rate, and nevertheless the knock-on electron can be registered.

A similar method was used by Backenstoss et al.[2], who didn't use a spark chamber but a total absorption shower detector consisting of interleaved layers of iron and plastic scintillators. The field strength in the iron plates was $\pm$ 2 x 10^4 gauss; the measurement was done in a collimated μ-beam of 8 $\pm$ 15% BeV/c and an intensity of 2x10^3 particles per pulse of the CERN-synchrotron.

II. - THEORETICAL DISCUSSION

1) Fundamental considerations

The theoretical cross section for the scattering of polarized μ-mesons on polarized electron is given[3] by

$$(1) \qquad \frac{\partial \sigma}{\partial \varepsilon} d\varepsilon = \frac{\partial \sigma_0}{\partial \varepsilon} d\varepsilon - P_e P_\mu \cdot \frac{\partial \sigma_1}{\partial \varepsilon} d\varepsilon$$

with

$$(2) \qquad \frac{\partial \sigma_0}{\partial \varepsilon} = 2\pi r_0^2 \frac{m_e}{\beta^2 \varepsilon^2}\left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_m} + \frac{\varepsilon^2}{2 E_\mu^2}\right)$$

and

$$(3) \qquad \frac{\partial \sigma_1}{\partial \varepsilon} = 2\pi r_0^2 \frac{m_e}{\beta^2 \varepsilon^2} \times \frac{\varepsilon}{E_\mu}\left(1 - \frac{\varepsilon}{\varepsilon_m} + \frac{\varepsilon}{2 E_\mu}\right)$$

Here we have taken:

m_e = rest energy of the knock-on electrons
β = velocity of the incident muons
r_0 = classical electron radius
$E_\mu = T_\mu + m_\mu$ = total energy of the incident muons
ε = energy transfer from μ to e^-
ε_m = maximum possible energy transfer
$\mathcal{P}_{e,\mu}$ = polarization vectors of e^-, μ , respectively.

From the kinematics of relativistic particles we find for the maximum possible energy transfer from an incident μ -meson to an electron:

(4) $$\varepsilon_m = \frac{E_\mu^2 - m_\mu^2}{\frac{1}{2} m_e \left\{ 1 + \left(\frac{m_\mu}{m_e} \right)^2 \right\} + E_\mu}$$

and for the connection between ε and the scattering angle ϑ of the knock-on electron, measured with respect to the primary direction of the incident μ -mesons:

(5) $$\sin^2 \vartheta = \frac{1 - \frac{\varepsilon}{\varepsilon_m}}{1 + \frac{\varepsilon}{2 m_e}}$$

Substituting

(6) $$\epsilon := \frac{\varepsilon}{\varepsilon_m} \quad ; \quad a := \frac{1}{2} \frac{\varepsilon_m}{E_\mu} \quad ; \quad \zeta := \frac{\varepsilon_m}{2 m_e}$$

we find from the integration of equations(2) and (3):

(7) $$\sigma_0(\epsilon) = A(T_\mu) \left\{ (1-\epsilon)\left(2a^2 + \frac{1}{\epsilon}\right) - \beta^2 \ln \frac{1}{\epsilon} \right\}$$

and

(8) $$\sigma_1(\epsilon) = A(T_\mu) \cdot 2a \left\{ \ln \frac{1}{\epsilon} - (1-a)(1-\epsilon) \right\}$$

with

(9) $$A(T_\mu) := \frac{\pi r_0^2}{\gamma \beta^2}$$

In the present experiment we reverse the magnetization in the iron plates of the spark chamber to change the spin orientation of the knock-on electrons.

The flux of registered knock-on electrons at field "up" and "down" may be then $N_\uparrow$ and $N_\downarrow$ respectively.

Now we define as a characteristic quantity

(10) $$S := \frac{N_\downarrow - N_\uparrow}{N_\downarrow + N_\uparrow}$$

Because of

(11) $$N_\downarrow = N_e N_\mu \, l \, (\sigma_0 + P_e P_\mu \sigma_1)$$
$$N_\uparrow = N_e N_\mu \, l \, (\sigma_0 - P_e P_\mu \sigma_1)$$

with N_e = spatial density of electrons (e^-/cm^3)

N = frequency of μ-mesons (μ/sec GeV/c)

l = thickness of scattering material (cm)

we find immediately

(12) $$S = P_\mu P_e \cdot \frac{\sigma_1}{\sigma_0}$$

Using equations (7) and (8):

(13) $$S = P_\mu P_e \cdot 2a \frac{\ln\frac{1}{\epsilon} - (1-a)(1-\epsilon)}{(1-\epsilon)(2a^2+\frac{1}{\epsilon}) - \beta^2 \ln\frac{1}{\epsilon}}$$

The average value of S is to be found from

(14) $$\bar{S} = \frac{\int_{\epsilon_2}^{\epsilon_1} S(\epsilon)\, n(\epsilon)\, d\epsilon}{\int_{\epsilon_2}^{\epsilon_1} n(\epsilon)\, d\epsilon}$$

with

(15) $$n(\epsilon)\,d\epsilon := \frac{\partial \sigma_0}{\partial \epsilon}\,d\epsilon \cdot N_e N_\mu \ell$$

to be

(16) $$\bar{S} = \frac{2a\, P_\mu P_e A(T_\mu)\int_{\epsilon_2}^{\epsilon_1} s(\epsilon)\,d\epsilon}{\sigma_0(\epsilon_1) - \sigma_0(\epsilon_2)}$$

where we introduced the definition[x]

(17) $$s(\epsilon) := \frac{\left\{\ln\frac{1}{\epsilon} - (1-a)(1-\epsilon)\right\}\left\{1-\beta^2\epsilon + 2a^2\epsilon^2\right\}}{\left\{(1-\epsilon)(2a^2+\frac{1}{\epsilon}) - \beta^2 \ln\frac{1}{\epsilon}\right\}\epsilon^2}$$

It is important to notice that the characteristic quantity S, defined by equation (10), must vanish, if the incident μ-meson beam contains the same number of positive and negative μ-mesons.

A measurement in cosmic rays without helping of an analyzer magnet[1] obviously is possible only because of the positive excess of the muons in the atmosphere.

For the numerical calculation of $\bar{S}$ the integral over the function s(E) must be calculated graphically taking a certain ϵ-region given by the measuring arrangement.

For the limiting case of all electrons getting maximum energy transfer, we find with the definition

(18) $$q(\epsilon_1, \epsilon_2) := \frac{\bar{S}}{P_\mu} \qquad at \quad \epsilon_2 = 1$$

(x) - Notice: we notice that the limiting value of $s(\epsilon)$ for all knock-on electrons having maximum possible energy, that is $\epsilon_1 \to 1$, is

$$\lim_{\epsilon_1 \to 1} s(\epsilon) = a \quad , \quad at \;\; \epsilon_2 = 1.$$

(19) $$\lim_{\epsilon_1 \to 1} q(\epsilon_1) = P_e \left(1 + \frac{1}{2a^2 \gamma^2}\right)^{-1} = q(\epsilon)_{max}$$

with $$\gamma := (1-\beta^2)^{-1/2}$$

Now we wish to determine the time t necessary to obtain the quantity S (see eq. (10)) with a relative error of $\pm$ p%.

That is, supposing gaussian distribution of $N_{\downarrow}$ and $N_{\uparrow}$, we demand a number n_e of knock-on electrons

(20) $$n_e \equiv t \cdot (N_{\downarrow} + N_{\uparrow}) \geq (p\, S)^{-2}$$

If we take the relative number of knock-on electrons per incident muons as $k \cdot \ell$, we find from equation (11) and (20) and the definition of N_{μ}

(21) $$k(T_{\mu}) := 2\, N_e \left\{ \sigma_0(\epsilon_2) - \sigma_0(\epsilon_1) \right\}$$

for measuring in a certain region of energy-transfer

$$\epsilon_1 \leq \epsilon \leq \epsilon_2 \; .$$

Thus it follows from eq. (20)

(22) $$t \geq \left[q^2 p^2 P_{\mu}^2\, k\, \ell\, N_{\mu} \right]^{-1}$$

In this expression k and q are functions of the initial energy of the incident μ-mesons and of the usable energy-transfer region determined by the measuring arrange ment.

2) Numerical example

We now shall consider the case of monochromatic muons of initial energy

$$\left. \begin{aligned} T_{\mu}^{\circ} &= 3\ BeV \\ m_{\mu} &= 105{,}66\ MeV \end{aligned} \right\} \Rightarrow E_{\mu}^{\circ} = 3{,}106\ BeV$$

equation (4) ⟶ $\varepsilon_m = 0{,}683$ BeV

equation (6) ⟶ $a = 0{,}111$

$\zeta = 672$

$\beta^2 = 0{,}9988$

Constants:

$$m_p = 1{,}67 \cdot 10^{-24} \text{ g}$$

$$r_0 = 2{,}818 \text{ f}$$

Iron - scatterer: $P_e = 8\ \%$

$A = 56$

$Z = 26$

$\varrho = 7{,}88 \text{ g cm}^{-3}$

$$N_e = \frac{Z\varrho}{A m_p} = 2{,}2 \times 10^{24} \frac{\text{electrons}}{\text{cm}^3}$$

The numerical calculation of the foregoing equations gives with those values:

$$\sin\vartheta = \frac{1-\epsilon}{1+672\,\epsilon} \tag{5'}$$

A graph of this function is given in fig. 1.

The region of scattering angles ϑ usable by a spark chamber is about

$$1° \leq \vartheta \leq 30°$$

Thus from fig. 1 or equation (5') we see, that knock--on electrons with an energy transfer ϵ not bigger than 0,83 can be registered, if the electrons pass only one spark gap.

That is, the one integration limit in equation (16) is

$$\epsilon_2 = 0{,}83$$

If we assume that the knock-on electrons to be registered must have at least an energy of $\varepsilon = 20$ MeV, we find the other integration limit to be

$$\epsilon_1 = 0{,}029$$

The contribution of s($\epsilon > \epsilon_{?}$) is relatively small. Thus, remembering the uncertainty of the value $\bar{\epsilon}$ = 20 MeV, we can take $\epsilon_{\ell} = 1$, indipendent of the muon energy.

From numerical integration of equation (16) we find for that region

$$0{,}03 \leq \epsilon \leq 1$$

(18')
$$q\,(3\ \mathrm{BeV}) = \frac{\bar{S}\,(3\ \mathrm{BeV})}{P_\mu} = 3{,}37\ ‰$$

and

(21')
$$k\,(3\ \mathrm{BeV}) = 4{,}87 \times 10^{-2}\ \frac{\text{knock-on electrons}}{\mu\text{-meson} \cdot \text{cm Fe}}$$

For rough comparison with tha Data of CERN(2)

$N_\mu \approx 266\ \mu$/sec ; P_μ = 62%
$p \approx \pm$ 27,3% ; t = 12,5 h = 4.5x10^4 s

we find from equation (22) for the total absorption length necessary

$$l = 5{,}3\ \text{cm}$$

At the CERN experiment(2) the total absorption length was 20 cm in iron.

This comparison is not yet quite correct because of the higher energy of the incident muons used in CERN.

If we take T_μ = 8 BeV into our calculation we find

$$l = 10{,}2\ \text{cm}$$

Thus the new method seems to be better by a factor 2 than the method used in CERN.

A graph of the energy dependent term of the time equation (22), that is of

(23)
$$\eta := (k \cdot q^2)^{-1}$$

is shown in fig. 2.

3) Measurement in a monochromatic beam

If we measure in a monochromatic beam of μ -mesons, that is in a beam containing a certain momentum band

$$p_i \leq p \leq p_f$$

we have to take the average value of the measuring time from the following formula:

$$(24) \qquad \bar{t}_{min} = \frac{\int_{p_i}^{p_f} t_{min} N_\mu \, dp_\mu}{\int_{p_i}^{p_f} N_\mu \, dp_\mu} ,$$

where t_{min} is the expression (22).

Setting

$$(25) \qquad N_\mu = \varkappa_\mu \cdot \Omega \cdot F$$

we get from equations (22) and (23):

$$(26) \qquad \bar{t}_{min} = \frac{1}{(p P_\mu)^2 \ell \Omega F} \cdot \frac{\int_{p_i}^{p_f} \eta \, dp_\mu}{\int_{p_i}^{p_f} \varkappa_\mu \, dp_\mu} .$$

III. - EXAMINATION OF THE POSSIBILITY OF A SIMPLE EXPERIMENT USING COSMIC RAYS FOR MEASURING THE POLARIZATION OF μ -MESONS

As mentioned above the method discussed can be used with cosmic rays only because there is a bigger number of positive muons than negative ones ("positive excess"). But the appearance of muons of both charges in the beam makes the effective counting rate less. That is, we get a longer measuring time than in the case of a "pure" beam.

We shall discuss at first the measuring time we have to expect measuring with a spark chamber in the open air. Later we shall see if it is possible to shorten the measuring time using a magnetic separator and measuring in a "pure" beam of cosmic rays. Finally we shall discuss a certain arrangement of two spark chambers, called a double spark chamber device.

1) The influence of the positive excess

Let n_μ^+ and n_μ^- be the numbers of positive and negative muons in the cosmic ray beam. Then positive excess means

$$n_\mu^+ = (1 + \delta)\, n_\mu^- \tag{27}$$

where δ is about 20%[4]

In equation (10) we defined as characteristic quantity of the measurement

$$S := \frac{N_\downarrow - N_\uparrow}{N_\downarrow + N_\uparrow} \quad . \tag{10}$$

In a symbolism easy to understand we can write for the numbers of knock-on electrons created by μ^+, μ^- respectively with magnetic field direction "up" and "down":

$$\left.\begin{array}{l} N_\downarrow (\mu^+) =: N_\downarrow^+ \\ N_\downarrow (\mu^-) =: N_\downarrow^- \\ N_\uparrow (\mu^+) =: N_\uparrow^+ \\ N_\uparrow (\mu^-) =: N_\downarrow^- \end{array}\right| \quad \begin{array}{l} N_\uparrow = N_\uparrow^+ + N_\uparrow^- \\ N_\downarrow = N_\downarrow^+ + N_\downarrow^- \end{array} \tag{28}$$

Now equation (27) is valid, and the reaction of μ^+ at field "up" is equal to that of μ^- at field "down" and viceversa. For this reason we get as a result of the measurement not S but the smaller quantity

$$S^{+-} := \frac{\delta}{2+\delta}\, S \quad . \tag{29}$$

To measure the polarization again with a relative error of p% we now find instead of equation (26) for the measuring time

$$\bar{t}^{+-} = \left(\frac{2+\delta}{\delta}\right)^2 \times \bar{t} \approx 120 \times \bar{t} . \tag{30}$$

2) Direct measurement using cosmic rays

If we consider muons passing 30 cm of iron to be registered, the low momentum end of the spectrum will be about

$$p_i = 500 \text{ MeV/c}$$

The high momentum end will be in any case about

$$p_f = 10 \text{ BeV/c} \quad ,$$

because muons of higher momentum will be scattered at a too small scattering angle ϑ to be registered (see fig. 1).

In the region $3 \leq p_\mu \leq 10$ BeV/c the spectrum of cosmic μ -mesons can be described[4] by a function

$$\varkappa_\mu (p) = a \times p_\mu^{-n} \tag{31}$$

with $a = 5{,}19 \times 10^{-3} \frac{\mu \text{ mesons}}{\text{sec cm}^2 \text{ sterad}}$

and $n = 1{,}83$

Therefore we divide the integral in equation (26) into a sum of two integrals. The first part, from 0,5 to 3 BeV/c, we have to calculate graphically, in the second part we shall use equation (31). From this we find

$$\int_{0,5}^{10 \text{ BeV/c}} \varkappa \, dp = 5{,}89 \times 10^{-3} \frac{\mu\text{-mesons}}{\text{sec sterad cm}^2} \tag{32}$$

This number is in good agreement with the total muon-flux at sea level, referred by Rossi[5].

The graphical integration of the other integral gives

$$\int_{0.5}^{10 \text{ BeV/c}} \eta \, dp = 3{,}04 \times 10^{6} \text{ cm BeV} \tag{33}$$

From this we find the measuring time:

$$(34)\quad \left(\bar{t}^{+-}\right)_{0.5}^{10\ BeV/c} = \frac{6,24 \cdot 10^{10}\ cm^{3}\ sterad\ sec}{(\not{p}\ P_{\mu})^{2}\ \ell\ \Omega\ F}$$

Let us use for determining the solid angle Ω of the measuring arrangement two rectangular scintillation counters with useful area F = 30 x 30 cm^2 being in a distance of 60 cm one from another.

Then the solid angle will be nearly,

$$\Omega \approx 0,7\ sterad$$

Supposing the values

P_{μ} = 50%

$\not{p}$ = 30% we get from eq. (34)

d = 20%

ℓ = 30 cm$_{Fe}$ $\bar{t}^{+-}_{min} \approx 4,7$ years.

Thus it is impossible to make this experiment with such a simple device.

3) Measurement using a magnetic separator (see fig. 3)

In the last section it was shown, that the measuring time in an experiment using cosmic rays is too long because of the smallness of the positive excess. Now we shall show that it is possible to get a shorter measuring time using a "pure" beam of cosmic muons separated by a magnetic separator. The disadvantage of this method is the small solid angle and the energy-loss of the incident muons in the iron of the magnet.

We shall consider a certain magnet available in this laboratory. This magnet gives a homogeneous field B of 13 kG, if the space between the poles is filled with iron. The proportions may be the following:

- height X (direction of incident μ -meson)

- depth Y
- width Z (field direction)

In a magnetic field B the radius of curvature of a particle with charge e and momentum p is given by

(35) $$\varrho = \frac{p}{eB}$$

In our case this magnitude is not a costant but changes with x because of the energy loss of the muons in the iron.

For our purpose it is enough to consider a monochromatic beam of incident muons of both polarities. The kinetic energy may be 3 GeV. From energy-range tables(6) we see, that for this initial energy, the energy loss in about 1 m of iron gives

(36) $$p_\mu(x) = p_\mu^0 - 16{,}7\ \frac{MeV}{c} \cdot \frac{x_{Fe}}{cm}$$

if $p_\mu^0 = 3$ BeV/c

Thus, if we consider for a rough estimate only muons of this primary momentum, we can calculate approximatively with a mean value

$$\bar{\varrho} = 555 \text{ cm}$$

That means, for a rough estimate of the solid angle we can consider the particle tracks in the magnet iron to be straight lines.

The location of the spark chamber behind the magnet is determined by the edges of the μ^+ and μ^- beams respectively. Furthermore the enlargement of the beam width due to the multiple scattering in the iron(7) changes this location also.

The lateral displacement of a muon entering into the magnetic field in x-direction is in good approximation given by

$$(37) \quad y(X) = \frac{eBc}{p} \int_0^X dx \int_0^x \frac{dx}{E_\mu(x)}$$

if multiple scattering is neglected.

Corresponding to eq. (36) in the energy region

$$3\ BeV \geq E_\mu \geq 1,5\ BeV$$

the energy loss is given by

$$(38) \qquad E_\mu(x) = E_\mu^0 - 14,65 \cdot x\ \frac{MeV}{cm}$$

Using this we get from eq. (37):

$$(39) \quad y(X) = \frac{e_0 BX}{n} \left\{ 1 - \frac{E(X)}{mX} \ln \frac{E_0}{E} \right\}$$

giving y (100 cm Fe) $\approx$ 7,6 cm at T_μ^0 = 3 BeV and B = 13 kG. On the other hand without any magnetic field the lateral distribution(7) of such particles after traversing 100 cm of iron would be a gaussian curve with a half-width of $b_0 \approx$ 6,6 cm. Adding the two effects we get approximatively a displacement of that gaussian curve around the value of the lateral displacement y.

The angular distribution(7) of the outcoming μ-mesons is described also by a gaussian curve, if the magnetic field is neglected again. In this case, the half-width after 100 cm of iron is $2\omega \approx 3,6°$.

Because of this the beam width increases with the distance d from the magnet giving a half-width b of the lateral distribution curve of circa

$$(40) \quad b = b_0 + 2d\ tg\,\omega \approx \left(6,6 + \frac{\pi}{50}\frac{d}{cm}\right) cm.$$

For using the whole width of the spark chamber we have to locate it in a distance d from the magnet, which is found from some simple but laborious geometrical considerations to be

$$(41) \qquad d \approx u \cdot \mathrm{ctg}\, \psi - k$$

u and k being the width and length of the spark chamber respectively, and ψ the aperture in the x-y plane, in our special case found to be

$$\psi \approx 6{,}1°$$

In the x-z plane the usable aperture χ of the μ-beam is defined by the breadth and the length of the magnetic field and the spark chamber and by the distance d.

A geometrical consideration gives

$$(42) \qquad \mathrm{tg}\, \chi = \frac{1}{2} \frac{v + Z}{\sqrt{X^2 + Z^2} + d + k}$$

v being the breadth of the spark chamber.

In our numerical example

u = v = k/2 = 30 cm	$d \approx 220$ cm
X = Y = 30 cm	$\chi \approx 4{,}5°$
Z = 100 cm	$\psi \approx 6{,}1°$

we then find the solid angle of the arrangement to be

$$\Omega \approx \mathrm{arc\,tg}\, \chi \;\; \mathrm{arc\,tg}\, \psi = 8{,}4 \cdot 10^{-3} \text{ sterad}$$

Without the magnetic analyzer we had found

$$\Omega_0 \cdot \left(\frac{d}{2+d}\right)^2 \approx 5{,}8 \cdot 10^{-3} \text{ sterad.}$$

Thus it is obvious, that using the available magnet we can not improve the measurement with cosmic rays.

4) Measurement with a double spark chamber device

We now want to examine another arrangement for mea suring the quantity S, defined by equation (10), using cosmic rays. This device should have the advantage giving a relatively big solid angle but no decrease of the counting rate because of the smallness of the positive excess.

Two spark chambers enclosed in iron may be disposed consecutively, and the magnetic fields in their plates may have opposite directions, as shown in fig. 4. With this arrangement we register the two tracks of each particle crossing both chambers on the same photograph by help of two mirrors.

Let $N_{\downarrow}$ and $N_{\uparrow}$ be the numbers of knock-on electrons created in the first and the second chambers respectively. Then we shall get $N_{\downarrow} > N_{\uparrow}$ for muons of a certain sign and obviously $N_{\downarrow} < N_{\uparrow}$ for muons of the opposite sign. Thus from each photograph we get either the quantity $S(\mu^{+}) =: S+$ or $S(\mu^{-}) =: S^{-}$ with

$$\frac{S^{+}}{|S^{+}|} = -\frac{S^{-}}{|S^{-}|}$$

In this manner we can distinguish the two different polarization vectors $P_{\mu^{+}}$ and $P_{\mu^{-}}$ but without being able to relate these values to the positive or negative muons. This is now possible by counting the sums of all events in the first and the second spark chambers respectively. Because of the positive excess these two sums will be different. If, for instance, the bigger sum belongs to the first chamber, we know that positive muons give us $N_{\downarrow} > N_{\uparrow}$ and vice versa.

NUMERICAL EXAMPLE

The two spark chambers, each containing 9 iron plates of 30 x 30 cm^2 and 1 cm thickness, may be arranged as shown in fig. 4.

For getting magnetic saturation in the iron plates we have to take a magnetic field of a fieldstrength B $\approx$ 20 kG.

From the well known formula

$$B = \mu_0 \cdot \frac{2nJ}{l},$$

where n is the number of spires at one side, we find with

$$\left.\begin{aligned} \mu_0 &= 12{,}57 \times 10^{-7} \\ l &= 10 \text{ cm} \\ B &= 20 \text{ kG} \end{aligned}\right\} \quad 2nJ \approx 1{,}6 \cdot 10^5 \text{ Amperturns}$$

With square hollow-wires of 1 cm x 1 cm we can have about 400 Amperes. Thus

$$n \approx 200 \text{ turns},$$

giving a rectangular coil of sidelengths 10 cm and 20 cm respectively.

The solid angle is determined by the scintillation counters which trigger the spark chamber device. If we locate them immediately before the first and behind the second spark chambers within the iron blocks, we find a solid angle of about 0,56 sterad. The muons have to pass 65 cm of iron to be registered. Thus the low momentum end of the spectrum will be about

$$p_i = 920 \text{ MeV}/c$$

The calculation is analogous to that shown in section 2, giving a measuring time of

$$\bar{t}_{min} \approx 56 \text{ days}$$

assuming P_μ = 50% and p = 30%.

Thus this experiment may be done.

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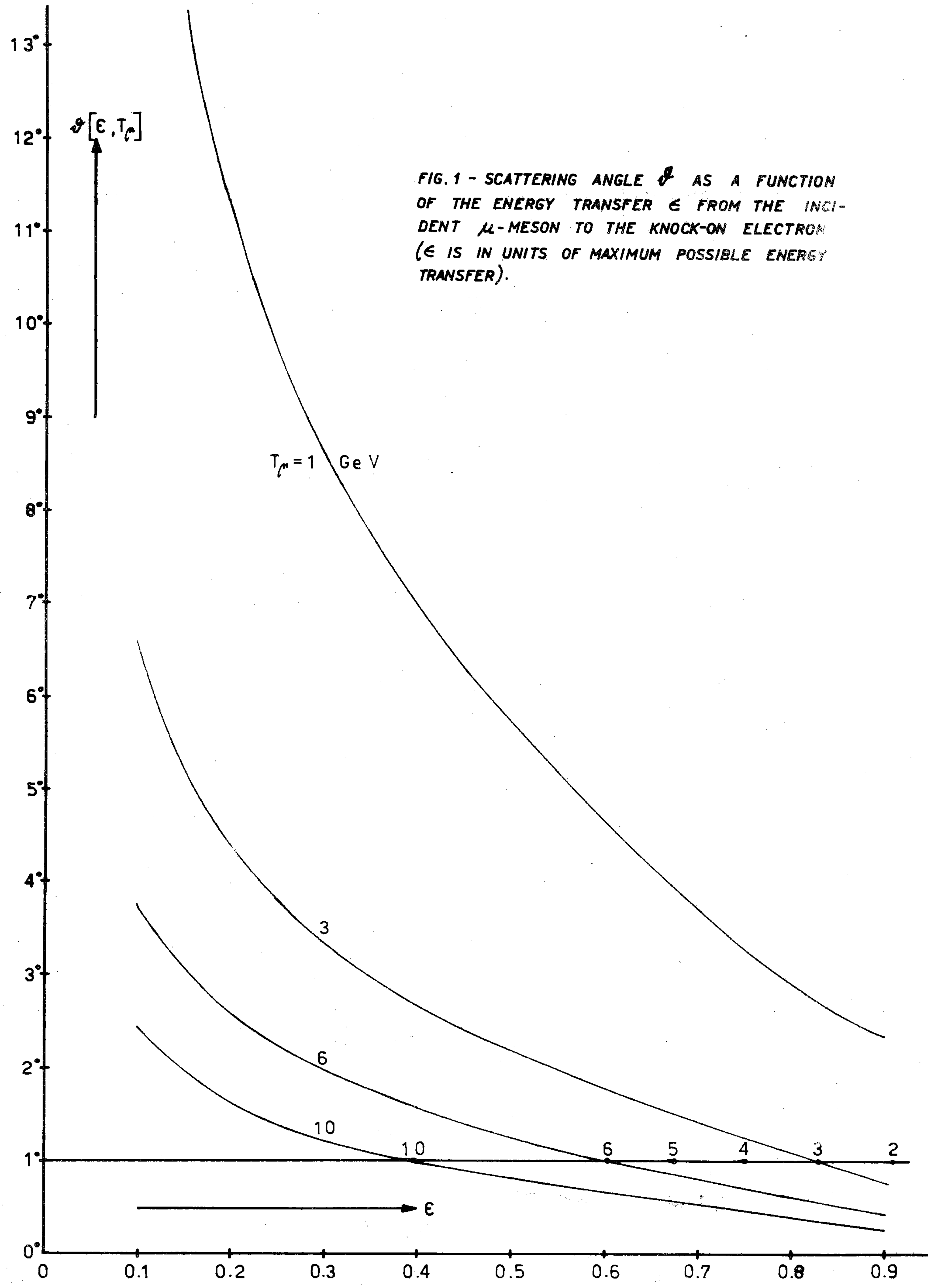

FIG. 1 - SCATTERING ANGLE ϑ AS A FUNCTION OF THE ENERGY TRANSFER ε FROM THE INCIDENT μ-MESON TO THE KNOCK-ON ELECTRON (ε IS IN UNITS OF MAXIMUM POSSIBLE ENERGY TRANSFER).

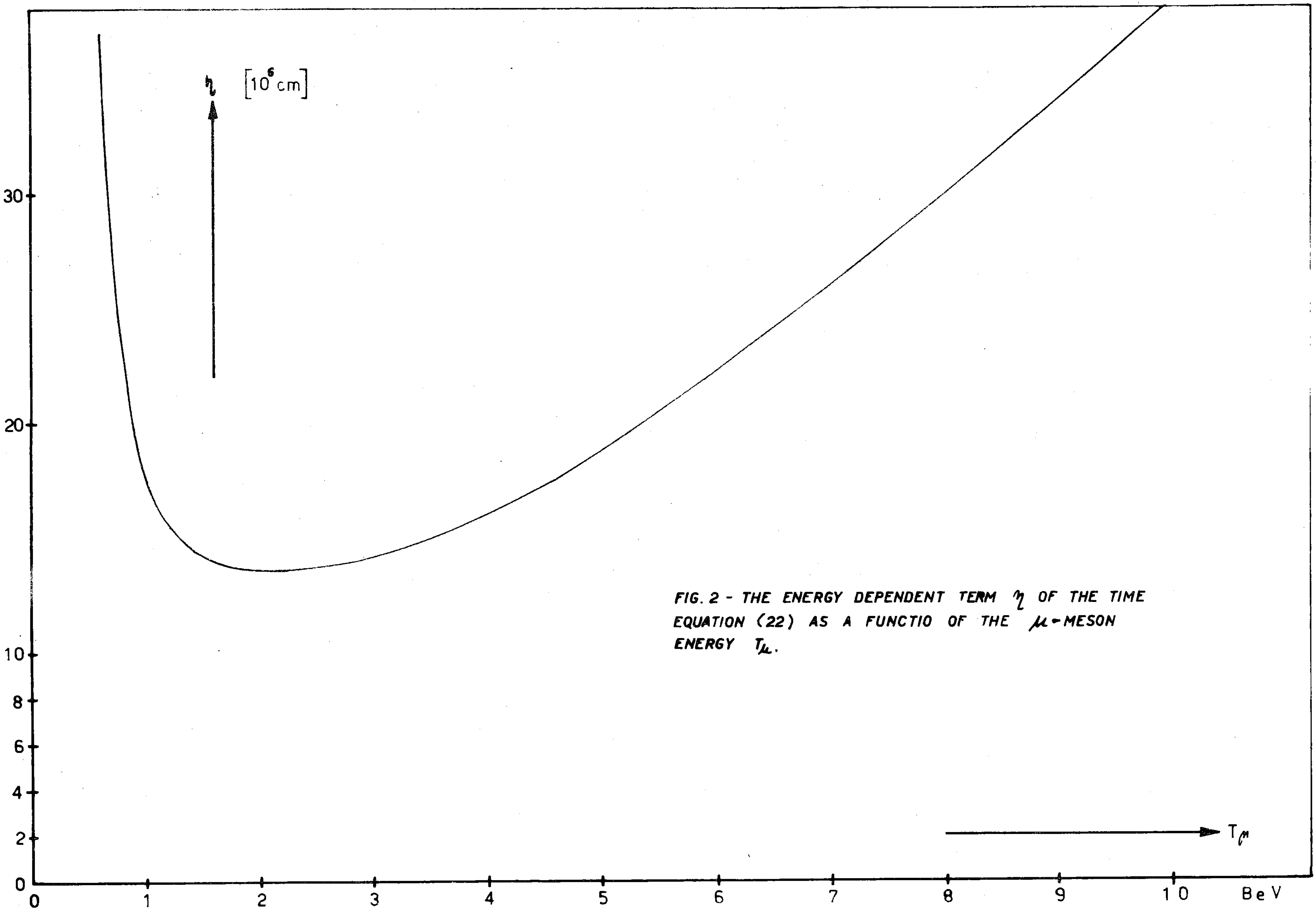

FIG. 2 - THE ENERGY DEPENDENT TERM η OF THE TIME EQUATION (22) AS A FUNCTIO OF THE μ-MESON ENERGY T_μ.

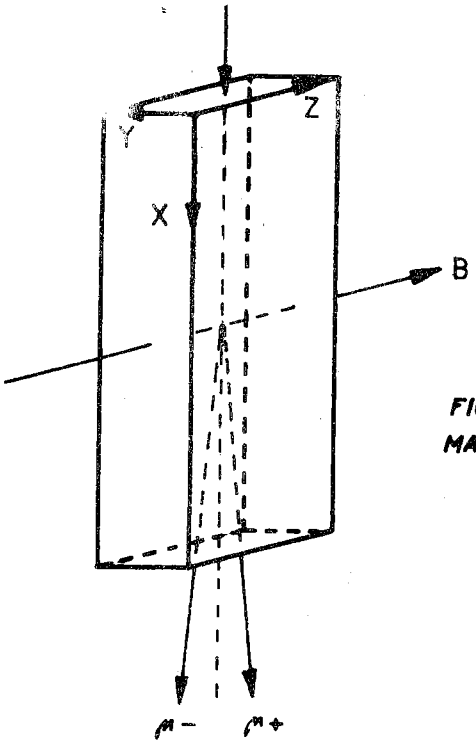

FIG. 3 - SKETCH OF THE MAGNETIC SEPARATOR

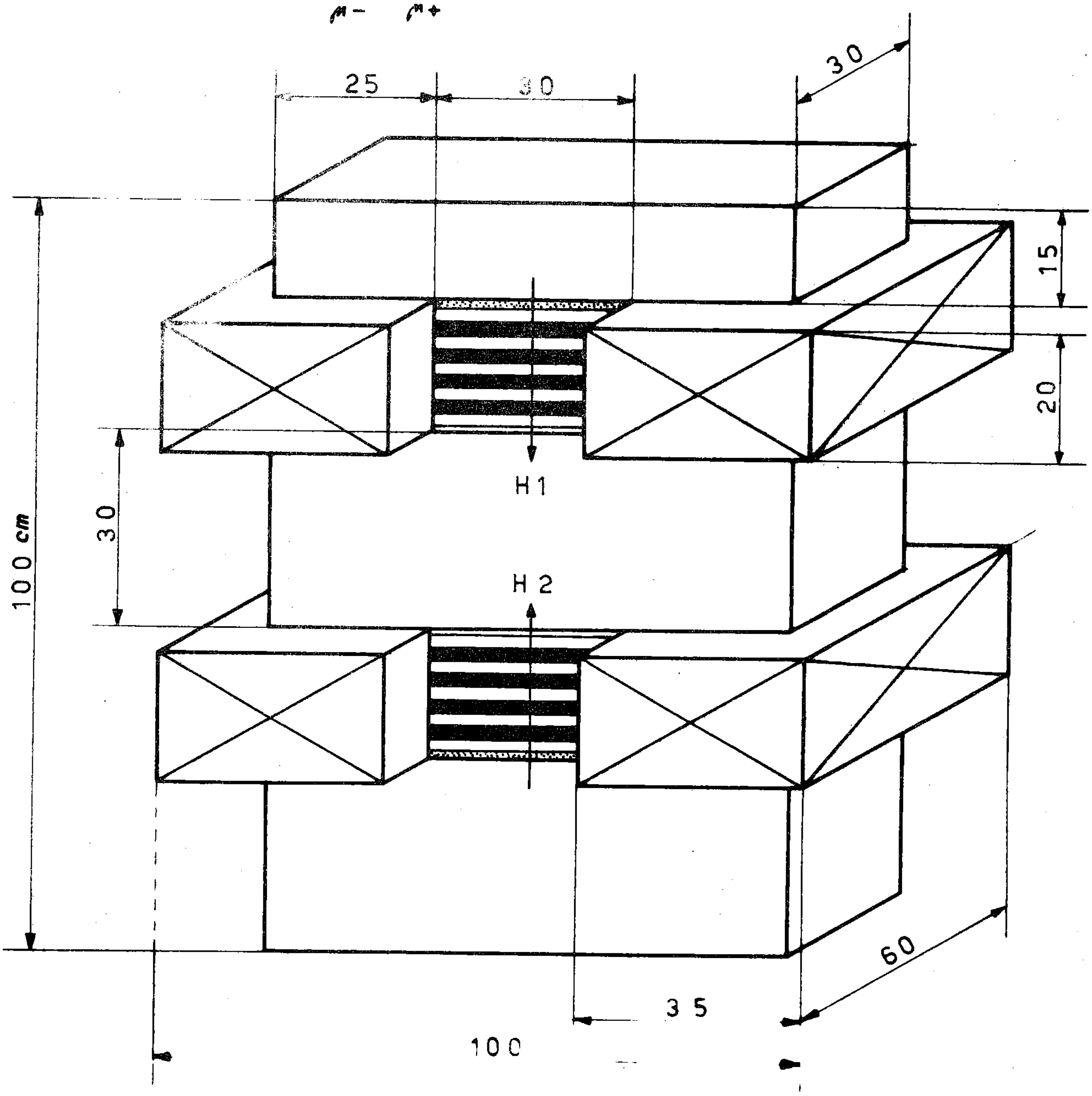

FIG [illegible] EVICE